

Supplement to:

Grow, André, Andreas Flache, and Rafael P. M. Wittek. 2017. "Global Diversity and Local Consensus in Status Beliefs: The Role of Network Clustering and Resistance to Belief Change." *Sociological Science* 4: 611-640.

## Appendix

In this appendix, we (1) explain under which conditions a simulation run can reach a stable equilibrium, so that no actor can be induced to change his/her status belief anymore, (2) explain in detail how the spatial distance parameter  $y$  relates to network clustering, and (3) discuss how we determined the number of simulation runs that were needed to obtain reliable results.

### Stable equilibria

In our model, it is possible that actor  $i$  is embedded in a configuration in which he/she cannot be induced to change his/her belief ( $S_i$ ) anymore, unless at least one other actor changes his/her state on  $S_i$  first. When all actors are in such a *locally stable configuration*, it is impossible that any actor will change his/her state on  $S_i$  in the future. In this case, we say that the simulation run has reached a *stable equilibrium*.

#### *Without belief inertia*

As Mark, Smith-Lovin, and Ridgeway (2009) have shown, when actors only consider their most recent interactional experience with somebody who differs from them in the social distinction ( $N_i$ ) in belief formation, the only stable equilibria are when all actors hold the belief  $S_i = A$  or the belief  $S_i = B$ . For this, it does not matter whether interactions occur in networks or not, or whether the networks are clustered or not.

#### *With belief inertia*

The situation changes when actors consider multiple experiences and can become resistant to belief change, in particular when interactions occur in network structures. For illustration, assume that more than 50 percent of an actor's most recent interactions with other actors who differ from him/her in  $N_i$  need to support a given belief to be acquired and maintained. Imagine that actor  $i$  currently believes  $S_i = A$  and is connected to the four other actors  $j$ ,  $o$ ,  $p$ , and  $q$ , who differ from  $i$  in  $N_i$ . These actors believe  $S_j = A$ ,  $S_o = A$ ,  $S_p = A$ , and  $S_q = B$  and  $i$ 's last interactions with these actors let to the memory  $M_i = \{A, A, A, B\}$  (ordered so that the elements refer to the interactions  $ij$ ,  $io$ ,  $ip$ , and  $iq$ ). In this situation,  $i$ 's experiences sufficiently support the belief  $S_i = A$ , so that he/she will maintain this belief, unless one of the first three elements of  $M_i$  changes. Such changes cannot happen as long as  $j$ ,  $o$ , and  $p$  also believe  $A$ . Thus, as long as  $j$ ,  $o$ , and  $p$  do not change their beliefs first,  $i$  cannot be induced to change his/her belief, no matter the outcome of his/her interactions with  $q$ . More generally speaking, we say that the

first three elements of  $M_i$  are *temporarily stable*, whereas the fourth element is *not stable*, because it could change to  $O$  or  $A$  during an interaction with  $q$ . Whenever the share of temporarily stable elements in an actor's memory with a status belief exceeds 50 percent (or 70 percent, or 90 percent, depending on which threshold is chosen), the actor cannot be induced to change his/her belief, unless some of his/her network neighbors who were involved in generating these stable elements change their states on  $S_i$  first.<sup>1</sup> Hence, the actor is in a locally stable configuration.

Also agents who currently hold no status belief can be in a locally stable configuration. Imagine that actor  $i$  currently holds no belief ( $S_i = O$ ) and, as above, is connected to the four other actors  $j$ ,  $o$ ,  $p$ , and  $q$  who differ from  $i$  in  $N_i$ . This time these other actors believe  $S_j = A$ ,  $S_o = A$ ,  $S_p = B$ , and  $S_q = B$  and  $i$ 's last interactions with each of them let to the memory  $M_i = \{A, A, B, B\}$ . In this situation, neither  $S_i = A$  nor  $S_i = B$  is sufficiently supported for  $i$  to acquire it. At the same time are all elements of  $M_i$  temporarily stable. The reason is that interactions with  $j$  and  $o$  will always support  $S_i = A$ , whereas interactions with  $p$  and  $q$  will always support  $S_i = B$ . Hence, unless some of  $i$ 's interaction partners change their beliefs first,  $i$  will not acquire a belief. More generally speaking, agents who hold no status belief can be in a locally stable configuration if

- (1) neither the share of temporarily stable elements in  $M_i$  that support  $S_i = A$  and the non-stable elements that could support this belief in the future together account for more than 50 percent (or 70 percent, or 90 percent) in  $M_i$ ,
- (2) nor the share of temporarily stable elements in  $M_i$  that support  $S_i = B$  and the non-stable elements that could support this belief in the future together account for more than 50 percent (or 70 percent, or 90 percent) in  $M_i$ .

Thus, when actors consider multiple experiences and interactions occur in network structures, it is possible that diversity in status beliefs emerges and remains in a stable equilibrium. This is also possible when agents consider multiple experiences but interactions are not constrained by network structures. In this case, any two members of the population can interact, so that  $M_i$  can contain as many elements as there are other actors who differ from  $i$  in

---

<sup>1</sup>  $S_i$  could also change when the number of elements in  $M_i$  would increase, because actor  $i$  interacts with new interaction partners with whom he/she has not interacted before. Thus, technically, we initialize agents' memory to have as many elements as there are actors with whom they can interact and who differ from them in  $N_i$ ; elements that refer to potential experiences with actors with whom a given actor could interact but has not done so yet are left empty and are considered not stable.

$N_i$ . This means that we always need to consider the belief distribution in the entire population to assess whether a given actor is in a stable configuration. For example, when exactly 50 percent of all those who belong to category  $N_i = A$  would believe  $S_i = A$ , whereas the other 50 percent believe  $S_i = B$ , and all actors who belong to category  $N_i = B$  would hold no belief ( $S_i = O$ ), it would be possible that no actor could be induced to change his/her belief anymore. However, such a situation has a very low probability to occur.

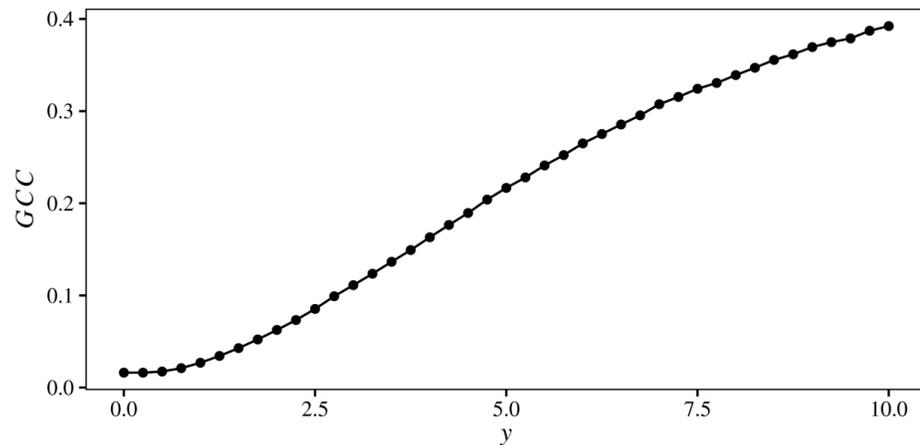
### The parameter $y$ and network clustering

Our first extension of the model proposed by Mark et al. (2009) imposes a network structure on the actor population. As we indicate in the main part of the article, the number of ties that exist in a given actor population is governed by the parameter  $k$ , and the level of network clustering is governed by the parameter  $y$ , which is used in the spatial distance function (Equation (1) in the main part of the article):

$$f(y, d_{ij}) = \exp(-y[d_{ij}]).$$

The outcome of this function is used to determine the probability with which a given actor  $i$  creates a tie to another actor with whom he/she is not connected yet. This function has the effect that actors are more likely to select those other actors as network partners who live close to them in physical space (if  $y > 0$ ). This means that ties become—in the first instance—clustered in physical space. As indicated in the main part of the article, this spatial clustering of ties also leads to network clustering, meaning that at higher values of  $y$  and a given value of  $k$ , the network neighbors of the actors tend to be more connected to each other. Here, we illustrate in more detail how  $y$  relates to a standard measure of network clustering, the global clustering coefficient  $GCC$  (as defined in endnote 2 in the main part of the article). We also discuss a complication that arises from the structure of the model proposed by Mark et al. (2009) and how we have addressed this complication.

To illustrate how  $y$  relates to  $GCC$ , we created networks according to the algorithm described in the main part of the article. For this, we considered  $I = 500$  actors, so that 250 belonged to category  $N_i = A$  and 250 belong to category  $N_i = B$ . We set  $k = 5$  and increased  $y$  from 0 to 10 in steps of 0.25. Given the stochastic nature of the network generation algorithm, we generated 50 independent networks for each level of  $y$  and averaged the resulting  $GCC$  values. Figure A1 shows the result of this analysis. For  $y = 0$ , the average value of  $GCC$  was about 0.016, which is equivalent to the value that is commonly obtained for networks in which

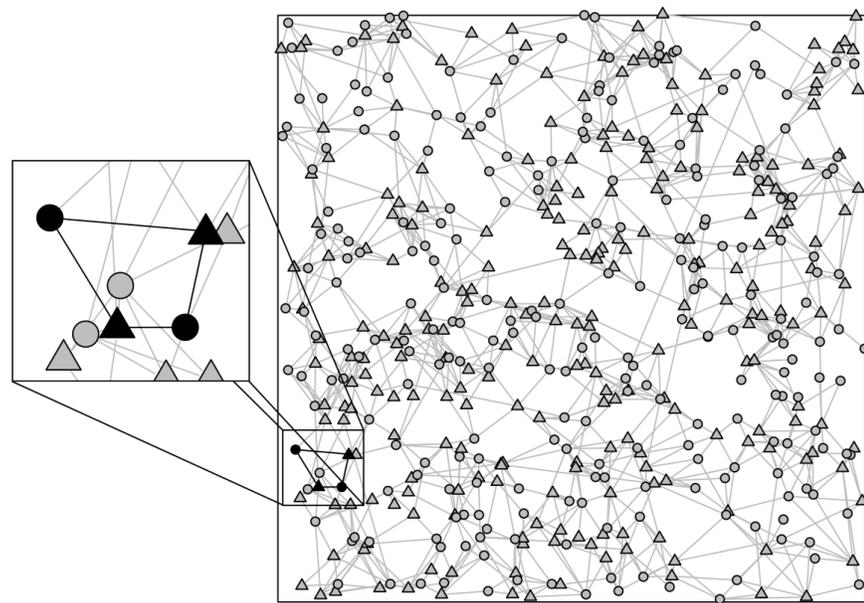


**Figure A1:** Association between the spatial distance effect  $y$  and the global clustering coefficient ( $GCC$ )

ties are generated completely at random (Watts and Strogatz 1998). As  $y$  increased, so did  $GCC$  in a monotonic fashion. For  $y = 10$ , the average value of  $GCC$  was about 0.4. Hence, in our model, the more the likelihood that actors are linked is affected by spatial distances, the more clustered the networks become.

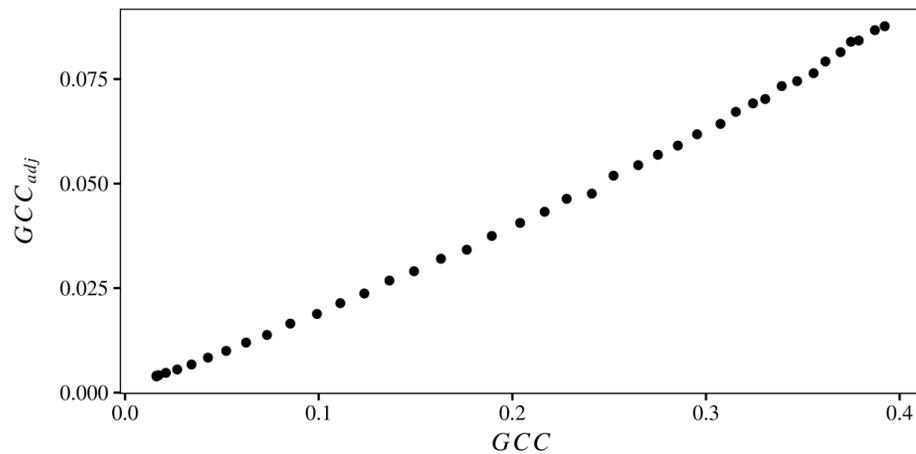
There is one complication that derives from the way in which Mark et al. (2009) implemented the belief formation process. In the original model, actors only infer differences in respect and competence from interactional experiences with actors who belong to a different category. That is, if actor  $i$  experiences that during one of his/her interactions with somebody who belongs to a different category of  $N_i$  one of them took the lower/higher status position,  $i$  uses this to inform his/her belief  $S_i$ . Hence, in the model of Mark et al. (2009), only interactions between actors who differ in  $N_i$  matter for belief formation and change, but not interactions between actors who belong to the same category of  $N_i$ . We followed this approach to modeling status construction processes in our second extension of the model and this has implications for our main argument.

Our main argument holds that network clustering can lead to the emergence and persistence of diversity in status beliefs, because when individuals consider multiple interactional experiences network clustering makes it likely that actors who share the same network neighbors will be exposed to similar experiences that can lead them to acquire similar beliefs. The resulting local consensus can then ward off influence from occasional



**Figure A2:** Example of network with clustering in which ties between actors who belong to the same category are removed with detailed view. Triangles and circles represent actors with  $N_i = A$  and  $N_i = B$ , respectively. Lines between actors indicate the possibility for interactions between them. The four actors indicated with black color belong to a 4-cycle relevant for calculating the adjusted version of the global clustering coefficient ( $GCC_{adj}$ ).

disconfirming experiences. Yet, given that in our model only interactional experiences with members of a different category of  $N_i$  affect actors' status beliefs, it does not only matter how clustered the network is per se. It also matters how much this clustering involves members of different social categories. More specifically, if we only focus on ties that connect actors who could affect each other's beliefs, the network becomes bipartite, because only ties between the members of the two different categories are relevant, but not ties between actors who belong to the same category. Figure A2 illustrates this with a network that was generated according to our standard algorithm, but from which we removed all ties between actors who belonged to the same category of  $N_i$ . Calculating  $GCC$  for this network yields 0, because none of the actors' direct neighbors can be connected to each other. For example, when  $i$  belongs to category  $A$ , the only connections that matter for his/her beliefs are to actors who belong to category  $B$ , who cannot be connected to each other. To assess the level of clustering that existed in these bipartite networks we developed a generalized version of the original  $GCC$  measure. The original  $GCC$



**Figure A3:** Association between the global clustering coefficient ( $GCC$ ) and its adjusted version ( $GCC_{adj}$ )

measure focuses on closed 3-cycles (e.g., actor  $i$  is connected to  $j$  and  $k$ , whereas  $j$  and  $k$  are also connected to each other, no matter their states on  $N_i$ ) and compares for each actor the number of such cycles of length 3 that have been realized with the number of cycles that could be realized. Technically,  $GCC$  is the ratio of the number of cycles of length 3 in the network to the number of paths of length 2. We adjusted this measure so that it focused on closed 4-cycles, so that it considered situations in which  $i$  was connected to  $j$ ,  $j$  to  $k$ ,  $k$  to  $q$ , and  $q$  to  $i$ , whereas the respective states on  $N_i$  of these actors were  $A, B, A$ , and  $B$  (or  $B, A, B$ , and  $A$ , see the detail view in Figure A2 for an example). These are the smallest possible cycles that are relevant for belief formation and maintenance in our model and our measure  $GCC_{adj}$  indicates the average share of the possible cycles of this type that were realized per agent. That is,  $GCC_{adj}$  is the ratio of the number of such 4-cycles to the number of similar paths of length 3 (i.e., paths in which the actor's states on  $N_i$  alternate) in the network. Figure A3 plots the average values of  $GCC$  and  $GCC_{adj}$  that we obtained for the same set of networks that underlie Figure A1 against each other. The two measures are positively and linearly related with each other. This means that as the 'traditional' form of network clustering increases, also the more specific form of clustering that is relevant for belief formation and maintenance in our model increases. Moreover, both forms of network clustering are associated in the same way with the spatial clustering coefficient  $\gamma$  that we use to vary network structures in our model.

### Determining the number of runs that are needed

Our simulation model is stochastic and outcomes can vary between simulation runs, even if they have the same starting conditions. We therefore averaged results across several runs to assess general tendencies in the model outcomes. Lorscheid, Heine, and Meyer (2012) proposed a measure of experimental variance (*CV*) to assess how many runs are needed to obtain reliable results, so that they are unlikely to change fundamentally if more runs would be added. The measure is calculated as the ratio of the standard deviation of a given outcome measure to its mean for a given set of runs. Increasing the number of runs typically increases the stability of the variance of the outcome and thus of *CV*. If further increasing the number of runs a given set does not change *CV* more than a predetermined criterion *E* anymore, the number of runs can be considered sufficient to obtain reliable results. Similar to Lee et al. (2015), we selected  $E = .01$ .<sup>2</sup>

To assess changes in *CV*, we used a separate set of simulation runs in which we gradually increased the number of runs and calculated *CV* after 10, 20, 40, 80, 160, and 320 runs. In this set, we chose all model parameters in the same way as in our main experiment reported in the main part of the article, assuming that interactions occurred in networks that were clustered and that belief inertia was possible. The corresponding *CV* values for *LS* were { .14, .16, .14, .14, .14, .14 } and for *NBS* { .13, .17, .15, .15, .15, .15 }.<sup>3</sup> Hence, after about 40 runs the changes in *CV* remained below *E* for both measures and we therefore chose 50 runs for our simulation experiments.

### References

- Lee, Ju-Sung et al. 2015. "The Complexities of Agent-Based Modeling Output Analysis." *Journal of Artificial Societies and Social Simulation* 18(4):4.
- Lorscheid, Iris, Bernd-Oliver Heine, and Matthias Meyer. 2012. "Opening the 'Black Box' of Simulations: Increased Transparency and Effective Communication through the Systematic Design of Experiments." *Computational and Mathematical Organization Theory* 18(1):22–62.
- Mark, Noah P., Lynn Smith-Lovin, and Cecilia L. Ridgeway. 2009. "Why Do Nominal Characteristics Acquire Status Value? A Minimal Explanation for Status Construction."

<sup>2</sup>Note that in the case of our model, calculating *CV* is only feasible for conditions in which belief inertia is possible, given that when there is no inertia, there is no variation in outcomes.

<sup>3</sup>We omitted *CS* given that this outcome measure is based on *LS*.

*American Journal of Sociology* 115(3):832–62.

Watts, Duncan J. and Steven H. Strogatz. 1998. “Collective Dynamics of ‘Small-World’ Networks.” *Nature* 393(6684):440–42.