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Supplement to:

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S1 Computational Analysis of Reflection Mechanism Implications

The reflection mechanism has been investigated under an unrealistic structural stability assumption (Friedkin, 2011; Jia et al., 2015). A stability assumption was invoked to focus the analysis on the implications of the evolution of $1 - a_{ii}(s) = w_{ii}(s)$. The stability assumption is that the adjustments of $1 - a_{ii}(s+1) = w_{ii}(s+1) = C_i(s)$ occur in a fixed strongly connected structure of relative interpersonal influences $\mathbf{R} = [r_{ij}]$ with $0 \leq r_{ij} \leq 1$ for all i and j, $r_{ii} = 0$ for all i, and $\sum_{j=1}^{n} r_{ij} = 1$ for all i. Thus, with a stable $\mathbf{R}$ and an evolving $\mathbf{A}$, we are presented with a $\mathbf{W}(s)$ that adjusts along the issue sequence strictly on the basis of individuals' self-weight adjustments,

$$\mathbf{W}(s) = \mathbf{A}(s)\mathbf{R} + \mathbf{I} - \mathbf{A}(s), \quad s = 1, 2, \ldots. \tag{1}$$

The Figure 1 illustration is based on the above stability assumption. Its $\mathbf{R}(s = 1)$ construct is a realization of strongly connected Gilbert directed random graph $G(n, p)$. Values, randomly drawn from the uniform distribution, are assigned to each edge of the graph and normalized to form $\mathbf{R}(s = 1)$. The main-diagonal values of $\mathbf{A}(s = 1)$ are randomly drawn from the uniform distribution. The $\mathbf{R}(s = 1)$ matrix is maintained along along the issue sequence as a fixed construct.

Here, we substantially advance the theory on the mechanism by allowing $\mathbf{R}$ to randomly vary along the issue sequence,

$$\mathbf{W}(s) = \mathbf{A}(s)\mathbf{R}(s) + \mathbf{I} - \mathbf{A}(s), \quad s = 1, 2, \ldots. \tag{2}$$

The Figure 2 illustration employs the same $\mathbf{R}(s = 1)$ and $\mathbf{A}(s = 1)$ of Figure 1 and new random values are assigned to its edges at each $s = 2, 3, \ldots$. The Figure 3 illustration employs the same $\mathbf{R}(s = 1)$ and $\mathbf{A}(s = 1)$ of Figure 1 and introduces additional instability. New realizations of strongly connected Gilbert directed random graph $G(n, p)$ with normalized random values drawn from the uniform distribution occur at each $s = 2, 3, \ldots$. Hence, in Figure 3 both the configuration of edges and their values are random variables under the constraint of a strongly connected configuration of edges. With this relaxation of the stability assumptions, and the introduction of random realizations of the $\mathbf{R}(s)$ along the issue sequence, it is not at all obvious that the reflection mechanism's implications are robust. The form and extent of structural instabilities in natural groups dealing with an issue sequence is unknown. The usual case is likely to be somewhere between the stability assumption of Figure 1 and the extreme instability assumption of Figure 3.

The following computation analysis indicates that the mechanism's implication is robust under the extreme instability conditions (described above) involved in the Figure 3 illustration. The behavior of $14,000$ realizations are evaluated on systems with n members in the $[4, 50]$ interval, $n =${4,8,10,12,14,16,18,20,25,30,35,40,45,50}. For each n, $1,000$ systems were constructed and their trajectories along $s = 1, 2, \ldots$ assessed with respect to whether or not each system trajectory converged to a control matrix $\mathbf{V}$ with a unique column of all 1s. Without exception, in all $14,000$ realizations, such convergence occurred. As expected, the random fluctuations of $\mathbf{R}$ in both configuration and values along the issue sequence induce random fluctuations of influence centrality values. However, in this structural noise, the reflection mechanism elevates the self-weight of one member toward 1 and the self-weights of the other $n - 1$ members toward 0. This differentiation dampens the salience of differences of influence allocation weights. That is, in a strongly connected influence network, with one nearly completely closed member and $n - 1$ members who are nearly completely open to influence, the particular configuration of allocations of influence and their values is irrelevant to the system's outcome. A near exact consensus will be generated on the initial opinion of the uniquely nearly closed member. The system repeatedly moves toward this state in the aftermath of a severe structural disturbance. Influence centralities may dramatically alter with structural disturbances. Such disturbances may reorganize the system and put a different member of the group on a trajectory toward maximal influence centrality.

S2 The DeGroot Attractor System

The evolution of influence allocations, based on the reflection mechanism, is toward a special case of a DeGroot influence system. In this section, we define the DeGroot attractor system. The form of the DeGroot (1974) system is

$$\mathbf{x}(k) = \mathbf{W}\mathbf{x}(k-1), \quad k = 1, \ldots, \tag{3}$$

It is an important system. Given all different initial opinions, $x_1(0) \neq x_2(0) \neq \cdots \neq x_n(0)$, reaching consensus can only occur with it. It appears as a special case of the Friedkin-Johnsen system when the main diagonal values of $\mathbf{A}$ are binary values; that is, when all group members are completely closed to influence, or completely open to influence, or when some are completely closed and all others are completely open. It also appears as the limit of the Friedkin-Johnsen system, under the constraint $\mathbf{0} < \mathbf{A} < \mathbf{I}$, when the extremal values are approached $a_{ii} \to 0$ or $a_{ii} \to 1$ for each i.

Under the condition of a strongly connected $\mathbf{AW}$, the reflection mechanism puts all group members on trajectories toward complete closure or openness to influence: one trajectory is toward a state of complete closure and $n-1$ trajectories are toward states of complete openness to influence. With one individual who is approaching near maximal closure to influence $a_{ii} \to 0$, and all others who are approaching near maximal openness to influence $a_{ii} \to 1$, the $\mathbf{AW}$ matrix of the influence system approaches

$$\mathbf{AW} = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ w_{21} & 0 & w_{23} & \dots & w_{2n} \\ w_{31} & w_{31} & 0 & \dots & w_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ w_{n1} & w_{n2} & w_{n3} & \dots & 0 \end{bmatrix},$$

and the control matrix $\mathbf{V}$ of the influence system approaches

$$\mathbf{V} = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 1 & 0 & 0 & \dots & 0 \\ 1 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \dots & 0 \end{bmatrix}.$$

That is, the influence centrality of one member approaches its maximal value, and the strongly connected network approaches a unilateral network in which the uniquely closed member influences (directly or indirectly) all other members. These other members are near maximally open to influence, and hence the outcome of the influence process is a consensus on the initial opinion of the closed member. So long as this state is maintained, the outcome on all issues will be the same—a convergence to the initial opinion of the closed member. The interpersonal weights $\mathbf{R}$ of the open members may alter from issue to issue, but so long as this generic structure is maintained the influence system of group is a DeGroot consensus production system with the same stable generic outcome.

S3 Choice Dilemma Issues, Influence Allocations, and Pressures

The issues involve opinions on the minimum level of confidence, that is a value in the $[0, 1]$ interval, required to accept a risky option with a high payoff over a less risky option with a low payoff. Each group considers 15 issues, randomly drawn without replacement from 18 choice dilemmas developed by Stoner (1961) and Wallach (1962). These 18 issues are:

> In the following situation, you are asked to choose one of two alternatives. One alternative involves greater risk than the other, while also offering a greater potential reward. Consider the alternatives. Then in-

dicate what probability of success would be necessary for you to choose the alternative which is potentially more rewarding, but which also carries a greater degree of risk.

Choice Dilemma Scenario

Using the scale below, CIRCLE the chance of success that you feel would be necessary before you would choose the option that carries the greater degree of risk, and a greater promise of reward.

To accept the course of action that involves greater risk, what is the MINIMUM chance of success you feel is needed:

0% 5% 10% 15% 20% 25% 30% 35% 40% 45% 50% 55% 60% 65% 70% 75% 80% 85% 90% 95% 100%

*A choice of 100% indicates an unwillingness to take any risk.

- Job Choice. Imagine that you are an electrical engineer. You may stick with your present job, rewarded at a modest but adequate salary (small risk), or alternatively may take a new job just as rewarding considerably more money but no long-term security (great risk).
- Heart Disease. You have just completed a visit to your family doctor, and then to a cardiac specialist. You have been told that you have a severe heart ailment. Due to your heart disease, you must drastically curtail your customary way of life (small risk). There is an alternative. There is a medical operation available that has the potential to bring about a complete cure of your heart ailment. However, the operation could prove fatal (great risk).
- Investment Choice. Imagine you want to invest some money you recently inherited. You may invest in secure "blue chip" low return securities (small risk), or alternatively in more risky securities that offer the possibility of large gains (great risk).
- Final Seconds' Decision. Imagine you are a captain of a college football team. In the final seconds of a game with the college's traditional rival, you may choose a play that is almost certain to produce a tied score (small risk), or alternatively a more risky play that would lead to sure victory if successful, sure defeat if not (great risk).
- Plant Location. Imagine you are the president of an American corporation that is about to expand. You may build a new plant in the United States where returns on the investment would be moderate (small risk), or may decide to build in a foreign country with an unstable political history where, however, returns on the investment would be very high (great risk).
- PhD's Dilemma. Imagine you are a college senior planning graduate work in chemistry. You may enter university X where, because of rigorous standards, only a fraction of the graduate students manage to receive the PhD (great risk), or may enter university Y which has a poorer reputation but where almost every graduate student receives the PhD (small risk).

- Chess Maneuver. Imagine you are a low ranked participant in a national chess tournament, playing an early match with a high ranked participant. You have the choice of attempting (great risk) or not trying (small risk) a deceptive but risky maneuver that might lead to quick victory if successful or almost certain defeat if it fails.

- Career Choice. Imagine you are a college senior with considerable musical talent. You must choose between the secure course of going on to medical school and becoming a physician (small risk), and the risky course of embarking on the career of a concert pianist (great risk).

- Prisoner's Dilemma. Imagine you are an American prisoner-of-war in World War II. You must choose between possible escape with the risk of execution if caught (great risk), and remaining in the camp where privations are severe and you are tortured (small risk).

- Run for Congress or Not. Imagine you are a successful businessman with strong feelings of civic responsibility. You must decide whether to run for Congress on the ticket of a minority party whose campaign funds are limited (i.e., you have to spend your own money) (great risk) or not to run for Congress (small risk).

- Physicist's Dilemma. Imagine you are a research physicist, just beginning a 5-year appointment at a university. You may spend the time working on a series of short-term problems, which you would be sure to solve but which would be of lesser importance (small risk), or on a very important but very difficult problem with the risk of nothing to show for his 5 years of effort (great risk).

- Marriage. Imagine you are engaged but you must decide, in the face of recent arguments with your partner suggesting some sharp differences of opinion, whether to get married (great risk) or not (small risk). Discussions with a marriage counselor indicate that a happy marriage, while possible, would not be assured.

- Dentist's Dilemma. Imagine you are a dentist. Recently you have a severe pain on yourself. You must decide whether to undergo an operation which would remove the pain if successful but would prevent you from continuing your dental practice if unsuccessful (great risk), or not to undergo such an operation and tolerate the pain (small risk).

- Vacation or Doctor. Imagine you are about to embark on a vacation trip but you experience severe abdominal pains. You must choose between disrupting your vacation plans in order to see a doctor (small risk) and boarding an airplane for a long overseas flight (great risk).

- Father's Choice. Imagine you are a parent and have recently received a promotion. You are considering spending some savings originally set aside for your child's college education on a family trip to Europe (great risk to your child) or not (small risk).

- Couple's Dilemma. Imagine you couple must choose between allowing a complicated pregnancy to continue with danger to the mother's life (great risk), and having the pregnancy terminated (small risk).
- Airplane Accident. Imagine you are involved in an airplane accident. You must choose between rescuing only your child (small risk) and attempting to rescue both your spouse and child with the realization that both will be lost if the attempt is unsuccessful (great risk).
- Sports Car Racing. Imagine you are a recently married young man with a pregnant wife. You have a hobby of sports car racing, which is somehow dangerous. You are deciding whether to give up your hobby (small risk) or not (great risk).

At the end of each issue discussion, each group member reported influence allocations as follows:

> This form is used to measure the amount in which each other member of the group has influenced your final opinion.
>
> STEP ONE:
>
> Imagine that you have been given a total of 100 chips and that each chip represents influence upon your final opinion. Divide these imaginary chips into two piles, Pile 1 and Pile 2, as follows.
>
> Pile 1: The extent to which the discussion of the issue influenced your final opinion
>
> __________
>
> Pile 2: The extent to which the discussion of the issue did not influence your final opinion
>
> __________
>
> Total 100 (The number of chips allocated should sum to 100)
>
> STEP TWO:
>
> Write the POS (A, B, C, or D) of the other three group members. Distribute the number of chips in Pile 1 (if any) among them. A person should not be allocated any chips if he or she did not influence your opinion. The number of chips allocated to a person should indicate the extent of their influence on your opinion.
>
> Group Member _____ Chips _____
> Group Member _____ Chips _____
> Group Member _____ Chips _____
> Total _____ (your Pile 1 number recorded in Step 1)
>
> Check that the allocated chips sum to the number of chips that you recorded for Pile 1.

Each group was placed under either a high pressure or low pressure condition over all 15 issues it considered as follows:

> We would like you to reach an agreement on this issue. If at the end of fifteen minutes there are remaining differences that you believe might be reconciled, you may have an additional five minutes for discussion. You may terminate the session at any time if you believe that the remaining differences of opinion cannot be reconciled. However, it has been our experience that most discussion groups are able to reach an agreement within the fifteen (plus an optional five) minute time frame.
>
> The discussion that you will have may or may not lead you to alter your First Opinion, and you may or may not come to an agreement. Any of these outcomes are OK with us. You will have fifteen minutes in which to discuss the issue. You may have an additional five minutes if you want them.

S4 Application to Groupthink

The theory and findings of this article bear on two lines of prominent work that point to conditions that concentrate influence and power. The first is based on Michels (1962) classic observation of an "iron law of oligarchy" that elevates the power of single individuals or a small subset individuals in the determination of collective decisions; see Friedkin's (2011) discussion of this application. The second is based on the Janis (1972; 1989) observation of a "groupthink" state in which collaborative decision making groups quickly converge to consensus. Here, we discuss the application to the groupthink literature, which is currently a prominent focus of interest and empirical work, and seek to advance a group dynamics approach to understanding the emergence of groupthink.

Groupthink refers to the circumstance in which a group consensus on an issue is reached quickly. On simple or routine issues, groupthink denotes a useful form of social cohesion. Such issues are dealt with expeditiously. On complex or novel issues, groupthink poses a potential hazard. Its negative connotation is linked to Janis' seminal analysis of the origins of faulty group decisions (Janis, 1972, 1989). Janis concluded that such decisions are related to a group's social structure. On a specific issue, this social structure may be defined by two constructs: group members' displayed initial opinions and their allocations of influence to other group members. The conditions, symptoms, and hazards of groupthink are related to these two constructs. The group's range of displayed initial opinions is an important constraint on decisions. The group's matrix of influence allocations is also important as the basis of opinion changes and the efficiency with which opinion differences are reduced. Given different initial opinions, the group may quickly or slowly settle on a particular initial opinion, a compromise opinion, or a set of unreconciled opinions. Janis associates the hazards of groupthink with a circumstance in which either (a) the group's range of displayed initial opinion does not include the consideration of options that should have been considered, or (b) the group's influence allocation matrix is such that a quick convergence to consensus is reached when a slower more careful deliberation would have delivered a better decision. In essence, the hazards of groupthink are associated with a maladaptation of group social structure to the issue-specific complexity of the issue being dealt with.

Groupthink is a palpable phenomenon that has been difficult to formalize and empirically investigate. While the term "groupthink" with its negative connotation is now widely employed to designate a particular type of group, the body of empirical work triggered by Janis presents mixed results on the conditions, symptoms, and hazards of groupthink (Whyte, 1989; Park, 2000; Aldag and Fuller, 1993; McCauley, 1998; Esser, 1998; Flowers, 1977; Callaway et al., 1985; McCauley, 1989). It does appear, as Janis concludes, that decision fiascos often involve groups with little initial opinion diversity, a directive leader, high social cohesion, and quick convergence to consensus. The problem, of course, is that the conditional probability of failure cannot be reliably obtained strictly on the basis of information in a sample of failures. Indeed, in a sample of groups that presents both decision successes and failures, the conditional probability of success for groups *with* groupthink traits may be *higher* than the conditional probability of success for groups *without* groupthink traits. The upshot is that the structural conditions of groups that enable an efficient and effective collective intelligence, which generates consensus on complex issues, are currently not well-understood.

The application and implications of the present work to the literature on groupthink are straightforward. Four variables have framed groupthink analysis: the size of the range of initial opinions considered by a group, the rapidity with which a group reaches consensus, the social cohesion of a group, and the presence in the group of a directive leader. Based on his study group decision fiascoes, Janis concludes that the hazard rate of flawed group decisions is elevated in groups that rapidly converge to a consensus on the basis of a limited range considered options, high social cohesion, and a directive leader. The theory and findings of this article bear on the each of four variables that been emphasized in the groupthink literature.

The range of displayed initial opinions is an important constraint on group decisions. This conclusion is secured by work independent of the Janis analysis. An initial consensus will be maintained. Barnlund (1959) reported that, in small groups assembled to solve problems of logic, an initial consensus was not questioned (the group moved on to the next problem) regardless of whether the consensus was correct or incorrect. Similarly, Thorndike (1938) found that an initial consensus was rarely modified regardless of whether the consensus was correct or incorrect; in his results, an initial consensus was modified in only 3 of 725 group problem-solving trials in which the group's judgment was correct, and in 1 of 263 trials in which the group's judgment was incorrect. Consensus is either assumed to be correct (whether or not it is) or satisfactory; in either case, it is deemed conclusive. In small groups assembled to discuss issues on which there is no correct position (issues of judgment), then the interpersonal influences that occur in a group discussion do not as a rule generate opinions that are outside of the range of the initial displayed positions on an issue. Friedkin and Johnsen (2011) find 1 violation in 96 size $n = 3$ group discussions and 1 violation in 250 size $n = 4$ group discussions. Violations are more frequent in $n = 2$ group discussions. In the present data on size $n = 4$ group discussions, a violation occurred in 8% of the 450 group discussions. In these studies, the sizes of ranges of initial opinions vary substantially. Hence, it is difficult to escape the conclusion that the range of a group's initial opinions is a significant constraint. Its foundation, we believe, is a convex combination information integration mechanism that is a heuristic automatic cognitive algebra of the brain. Such a mechanism cannot produce positions on issues that are outside the convex hull of initial positions. The circumstances under which violations occur are currently not well understood.

Quick convergence to consensus may be measured in real time. However, an understanding of the structural foundations of the influence system that affects rates of convergence is not a transparent matter. The structural conditions of a group's influence network determines the rate of convergence of the system to a state of settled opinion on an issue. The convergence (fast or slow) may be to a state of unreconciled disagreement, to a consensus on one of the initial displayed positions, or to a compromise position. The implications of fast or slow convergence depend on whether the issue under consideration is simple (routine) or complex (novel). In the domain of complex issues, it is plausible that fast convergence to consensus poses a hazard when individuals abandon their initial opinion positions without discussion. In the formalization on single-issue dynamics, such abandonment occurs with individuals who allocate no weight to their initial opinions and immediately shift to a position that is the weighted average of the displayed opinions of those persons to whom they have allocated influence. In this formalization, if individuals allocate some ongoing weight to their initial opinions, then an exact consensus cannot be achieved, but the influence process may slowly move all group members toward a near consensus and a unanimous consent on a position. If groups concentrate their allocations of influence on one member, e.g., one expert or member who controls valued material or symbolic rewards, then a convergence to that member's initial opinion may be rapid. However, we cannot in general deduce a system's rate of convergence to consensus, when the system's structure is consistent with the production of consensus, without attending to the idiosyncratic features of the influence structure.

Social cohesion, which Janis posits as a structural condition of groupthink, is an important general construct and has been specified in a variety of ways. It is, therefore, essential to define what exactly is being referred to when the term is used. We do not use the term "social cohesion" in the formalization. Instead, we define constructs of the formalization to characterize the influence structure of a group. The defined constructs are specific structural conditions of influence networks that have bearing on the behavior of the influence system. These constructs include group members' levels of openness to influence, the density of their nonzero allocations of influence to other group members, and the topological category of the configuration of the group's nonzero allocations of influence (strong unilateral, weak, or disconnected). These constructs have implications for the reduction of opinion differences, the speed of the reduction, and the type of group outcome—the adoption of one of the member's initial opinion, a compromise consensus, or a settled state of important unreconciled opinion differences.

With respect to directive leadership, the article presents a more general perspective than is considered in the groupthink literature. The presence of a directive leader, who controls valued material and symbolic resources and has a stubborn attachment to his or her own opinions, is a plausible basis of constraint on the expressed opinions in a group. If group members' allocations of influence are concentrated on such a leader, then the group discussion only serves to ratify the leader's opinion. However, Barnard's (1938) seminal statement on the conditions of authority emphasizes that authority exists only when individuals have allocated influence to a person in a nominal position of authority:

> Authority is the character of a communication (order) in a formal organization by virtue of which it is accepted by a contributor to or "member" of the organization as governing the action he contributes; that is, as governing or determining what he does or is not to do so far as the organization is concerned . . . . If a directive communication is accepted by one to whom it is addressed, its authority for him is confirmed or established. Disobedience of such a communication is a denial of its authority for him. Therefore, under this definition the decision as to whether an order has authority or not lies with the persons to whom it is addressed, and does not reside in "persons of authority" or those who issue these orders. (p.163).

Moreover, directive leadership has at least two forms only one of which is emphasized in the groupthink literature. It includes the obvious case and the case of an authority who (a) does not voice an opinion and (b) strongly enforces an architecture of group deliberation that requires a full display and vetting of group members' opinions on an issue. This second form (a reserved authority) is the more interesting of two. The work of this article suggests that along an issue sequence, a *natural* process of influence differentiation occurs that will put the group on a trajectory toward a concentration of influence centrality. Thus, a paradox emerges in which the reticence of the nominal leader of group encourages a competition (explicit or implicit) for influence among the other group members that inevitably elevates one member's self-weight and influence centrality, and dampens all other members' self-weights and influence centralities. The group is continually moving, amidst structural disturbances, toward an influence system in which one member's opinion determines the group's consensus on an issue.

Thus, we believe that the difficulty of theory development on groupthink may be addressed with a formalization of group dynamics that occur on specific issues along a sequence of issues. In a group that is dealing with a sequence of issues, the social structure of the group may *evolve*. In a particular issue domain, the group's range of displayed initial opinion may narrow along an issue sequence as the group evolves toward a normative mindset that constrains the expression of particular positions. In groups that begin with diverse premises on a class of issues, the evolution may be toward reduced initial opinion diversity. In a particular issue domain, the group's matrix of allocated influences also may alter along an issue sequence toward greater efficiency of consensus production. In groups that begin with slow careful deliberations of issues, the evolution may be toward quicker consensus production. Our theory suggests this is the case, and that group trajectory toward greater efficiency is associated with the *emergence* of a leader whose initial opinions on issues determine the group's decisions. In this way, the formalization is addressed to an important piece of the groupthink puzzle. It does not address whether the evolution of a group toward greater efficiency is associated with a decrease or increase of collective intelligence.

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