

Supplement to:

Breen, Richard, Carina Mood and Jan O. Jonsson. 2016.
“How Much Scope for a Mobility Paradox? The Relationship between Social and Income Mobility in Sweden.” *Sociological Science* 3: 39-60.

Full decomposition results

Table A1. Decomposition of income correlations and class according to three models of intergenerational income mobility, for sons and daughters, using EGP classes and micro-classes, respectively.

EGP Classes

<i>Parent Inc/class</i>	<i>Child Inc/class</i>	<i>Inc corr</i>	SONS				DAUGHTERS				
			<i>class- class</i>	<i>resid- resid</i>	<i>resid(p) class(c)</i>	<i>class(p) resid(c)</i>	<i>Inc corr</i>	<i>class- class</i>	<i>resid- resid</i>	<i>resid(p) class(c)</i>	<i>class(p) resid(c)</i>
Fath/Fath Mod 1	Empl/Own %	0.299	0.12	0.07	0.06	0.05	0.197	0.10	0.02	0.05	0.03
			0.39	0.23	0.20	0.18		0.49	0.12	0.26	0.13
Disp/Dom Mod 2	Empl/Own %	0.281	0.10	0.07	0.06	0.05	0.197	0.09	0.03	0.05	0.03
			0.37	0.25	0.21	0.18		0.43	0.17	0.27	0.13
Disp/Dom Mod 3	Disp/Dom %	0.224	0.07	0.07	0.04	0.04	0.155	0.06	0.04	0.04	0.02
			0.33	0.32	0.19	0.16		0.38	0.29	0.24	0.11

Micro-classes

<i>Parent Inc/class</i>	<i>Child Inc/class</i>	<i>Inc corr</i>	SONS				DAUGHTERS				
			<i>class- class</i>	<i>resid- resid</i>	<i>resid(p) class(c)</i>	<i>resid(c) class(p)</i>	<i>Inc corr</i>	<i>class- class</i>	<i>resid- resid</i>	<i>resid(p) class(c)</i>	<i>resid(c) class(p)</i>
Fath/Fath Mod 1	Empl/Own %	0.299	0.14	0.05	0.06	0.05	0.197	0.10	0.02	0.04	0.04
			0.46	0.17	0.19	0.18		0.48	0.11	0.21	0.19
Disp/Dom Mod 2	Empl/Own %	0.281	0.12	0.06	0.06	0.05	0.197	0.08	0.03	0.05	0.04
			0.41	0.20	0.22	0.17		0.40	0.18	0.24	0.19
Disp/Dom Mod 3	Disp/Dom %	0.224	0.08	0.07	0.04	0.04	0.155	0.05	0.05	0.03	0.03
			0.35	0.29	0.19	0.18		0.32	0.30	0.21	0.18

Note: Mod 1: Standard model
 Mod 2: Origin family model
 Mod 3: Gross model

Intergenerational class mobility as a function of the intergenerational income correlation

In the text we write income mobility as a function of class mobility because, although our analysis is not causal, it does seem more reasonable to think of income as following from class position, rather than *vice versa*. However, since we are concerned with the mathematical relationship between the two sorts of mobility, we could write M , the normed mobility table, as a function of $\text{cov}(Y_p Y_c)$, as follows.

We write the relationship between income and class as a set of regressions for parent's class and child's class respectively:

$$X_{ipj} = m_{pj} + c_{pj} Y_{ip} + u_{ipj} \text{ for } j=1, \dots, J, \text{ and}$$

$$X_{icj} = m_{cj} + c_{cj} Y_{ic} + u_{icj} \text{ for } j=1, \dots, J$$

Here i indexes parent-child pairs, and X_j denotes a dummy variable for occupation of the j^{th} class. We define \mathbf{c}_p as the vector of regressions coefficients, c_{p1} to c_{pJ} and likewise for \mathbf{c}_c . Define $\text{cov}(\mathbf{u}_p \mathbf{u}_c)$ as the matrix of pairwise covariances between, on the one hand, the u_{pj} terms and, on the other, the u_{cj} terms. The marginal distributions of class are denoted \mathbf{p}_p and \mathbf{p}_c . Then we can write

$$M = \{\mathbf{c}_p \mathbf{c}_c' \text{cov}(Y_p Y_c) + \text{cov}(\mathbf{u}_p \mathbf{u}_c) + \mathbf{c}_p \text{cov}(Y_p \mathbf{u}_c) + \mathbf{c}_c \text{cov}(Y_c \mathbf{u}_p)\} + \mathbf{p}_p \mathbf{p}_c$$

The term in curly brackets is the covariance between the origin and destination dummy variables, and we recover the joint probability distribution by adding to this the outer product of the marginal probability vectors. Although each of the components is clearly defined and interpretable, the fact that each of the four parts in the curly brackets is itself a $J \times J$ matrix means that, in contrast to the decomposition shown in the text, it is not possible to extract a single number telling us how much one kind of intergenerational mobility contributes to the other.

Linking social fluidity and the intergenerational income correlation

To investigate the relationship between social fluidity and the intergenerational correlation of incomes we need to simulate the effects of changing social fluidity on the pattern of intergenerational social mobility. To do this we first fit a saturated log-linear model to the father–son mobility table. The log odds ratios of the table can then be expressed as a function of only the interaction parameters of this model, λ_{ij} , as follows:

$$\log(\theta_{ii'jj'}) = \lambda_{ij} - \lambda_{ij'} - \lambda_{i'j} + \lambda_{i'j'}, \text{ for } i \neq i', j \neq j' \quad (\text{A1})$$

where θ denotes an odds ratio and i and j index the row and column locations of the cells involved.

We can multiply the interaction parameters by a common scalar, s , $s > 0$, to yield the scaled log odds ratio:

$$s \times (\lambda_{ij} - \lambda_{ij'} - \lambda_{i'j} + \lambda_{i'j'}) = s \cdot \log(\theta_{ii'jj'}) \quad (\text{A2})$$

The scaled odds ratio itself is equal to the original odds ratio raised to the power of s : $\theta_{ii'jj'}^s$. This holds for all possible odds ratios in the table. For $0 < s < 1$ the scaled odds ratios are uniformly shrunk towards 1; that is, as s approaches 0, θ approaches 1 (and so the log odds ratios approach zero). For $s = 1$ the scaling reproduces the original odds ratios, and for $s > 1$ the scaled odds ratios have larger absolute values than the original odds ratios.¹

Our next step is to generate hypothetical tables with the observed margins of the mobility table and the scaled odds ratios. We first generate a hypothetical table using the main effect parameters from the original model and the scaled interaction parameters. But since this table does not reproduce the original margin,² we use the Deming-Stephan algorithm to accomplish this. The final result is a table with the observed marginal distributions of father's and child's class and odds ratios which follow the same pattern as in the observed data (in the sense that

¹ The scaling requires that every cell has a non-zero interaction parameter and so the interaction terms must be formed using centered rather than dummy variable coding because the latter identifies the parameters by setting the interaction terms for certain cells to zero.

² The saturated model reproduces the observed marginal distributions, but these are functions of both the main effect and interaction parameters and so, when we scale the latter, the hypothetical table does not reproduce the margins exactly.

both tables display the same ratio of odds ratios) but which are uniformly stronger or weaker, depending on s , than in the original table.

We repeat this process, varying the value of s , and we use each of the hypothetical tables to compute that part of the father–child income correlation transmitted via the mobility table;

using our earlier notation this is given by $\frac{\mathbf{b}'_p M^s \mathbf{b}_c}{\sigma_p \sigma_c}$, where M^s denotes the mobility table

scaled using the value s . Notice that, because we preserve the original marginal distributions, we can vary this element of the intergenerational income correlation without affecting any of the other elements in equation (5), which also include father's or child's class.