sociological science

# Supplement to:
# Pontkes, Elizabeth G. and Michael T. Hannan. 2014. "An Ecology of Social Categories." Sociological Science 1: 311-343.

## Nonmonotonic Quantification

The analytic approach on which we build states (some) definitions, postulates, auxiliary assumptions, and propositions as formulas in the language of a nonmonotonic logic (Pólos and Hannan 2002, 2004).[1] In formal terms, models of arguments are given in terms of sequences of intensions of open formulas. The formal language represents default rules (rules with possible exceptions) and defines a new kind of quantifier, denoted by $\mathfrak{N}$. Formulas quantified by $\mathfrak{N}$ state what is expected to "normally" be the case (by default) according to the theory. The normal case is what we assume to be the case if we lack more specific information that overrules the default. The implications of a set of rules with exceptions, which we call provisional theorems, are the logical consequences of a stage of a theory. Provisional theorems have a haphazard existence: what can be derived at one stage of theory development might not be derivable in a later stage if more specific considerations are brought into the picture. The syntax of the language codes this difference by introducing a "presumably" quantifier, denoted by $\mathfrak{P}$. Sentences (formulas) quantified by $\mathfrak{P}$ are provisional theorems at a stage of a theory if they follow from the premises at that stage. The formal semantics of these quantifiers are spelled out in **?**.

Testing what follows from the premises in a stage of a theory in the nonmonotonic logic we use operates on representations of arguments in the form of "rule chains." The links in these chains are strict rules, definitions, auxiliary assumptions, and postulates. The chains start with the subject of the argument and terminate with the consequence to be derived. In nonmonotonic inference, different rule chains—each representing an argument embodied in the stage of the theory—might lead to opposing conclusions. The testing procedure determines whether any inference can be drawn at all and, if so, which one. Such testing requires standards for assessing whether a pair of relevant rule chains is comparable in specificity and for determining specificity differences for comparable chains. In the case of this article, the available premises and definitions all point in the same direction; we do not see any rule chains that point to opposing conclusions. Thus all that is required is that we establish a rule chain that connects the antecedent and consequent in a claimed theorem.

We use an informal sorting of variables (which are strictly speaking just pointers to objects or tuples in the domain of discourse). The predicates we use have the general form $\textsc{predicate}(l, x, y, t)$ and the variables have the same: $\sigma(l, x, y)$. Sometimes there is more than one of each "type" of variable. Throughout we use the variable $l$ (or $l_1, \ldots$) to refer to a label (associated with a concept), $x$ and $x'$ to refer to producers, $y$ to refer to the agent who does the perception, valuation, and labeling, and $t$ and $t$ (or $t_1, \ldots$) to refer to time points. (In the case of self-labeling, $x$ and $y$ point to the same entity, the producer that labels itself.)

[1] This version of nonmonotonic logic builds heavily on the Amsterdam school of dynamic logic, especially on Veltman's (**?**) update semantics.